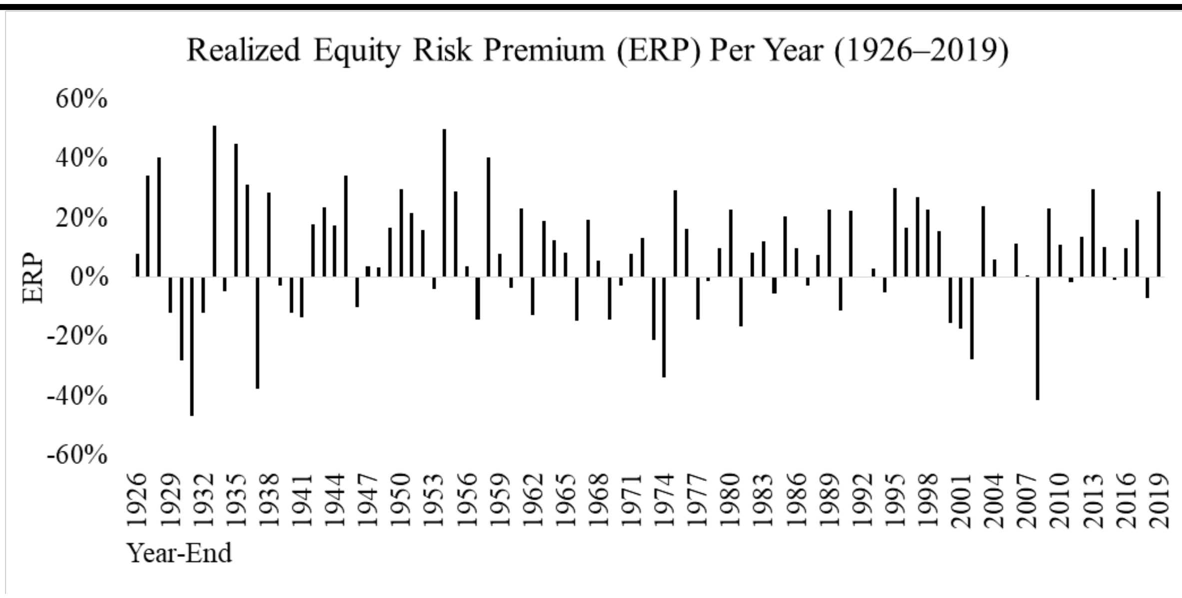
Using Arithmetic versus Geometric Mean of Historical Returns

The argument for using the arithmetic average is quite straightforward. In looking at projected cash flows, the ERP that should be employed is the ERP that is expected to be incurred over the future time periods. The graph in Exhibit A11–1 shows the realized equity risk premium for each year based on the returns of the S&P 500 and the income return on long-term government bonds. (The actual, observed difference between the return on the stock market and the riskless rate is known as the realized ERP.) There is considerable volatility in the year-by-year statistics. At times the realized ERP is even negative.

Exhibit A11–1

Arithmetic versus Geometric Mean for Calculating the Expected Equity Risk Premium for Estimating Discount Rate

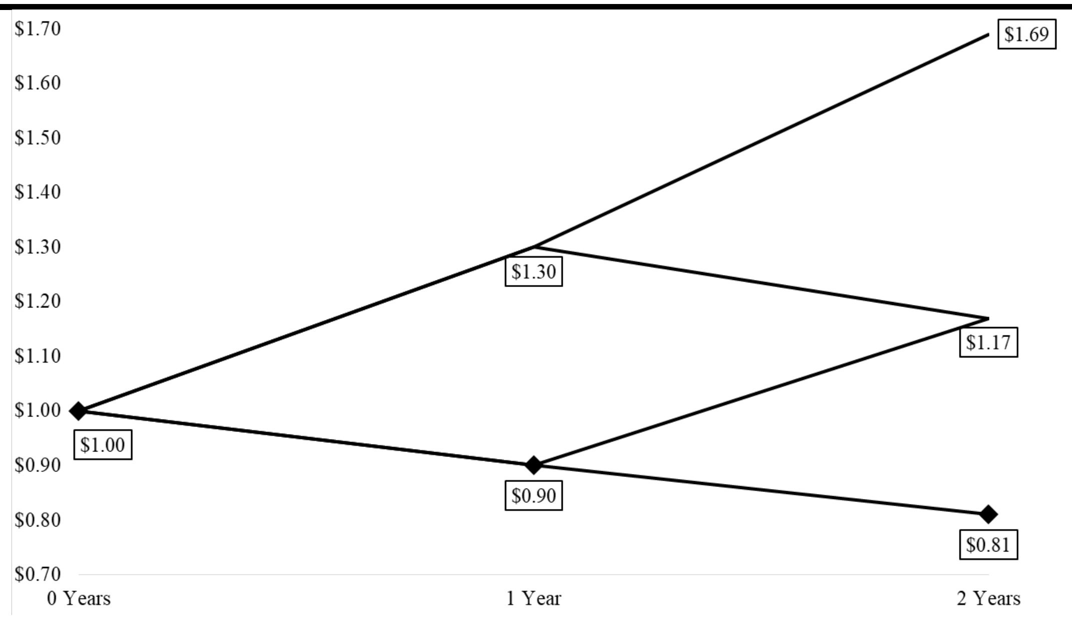


Source of underlying data: Morningstar. Exhibit 11–2 is constructed using the long-horizon "historical" ERP, as previously published in the former 1999–2006 Ibbotson Associates SBBI Valuation Yearbook, the former 2008–2013 Morningstar SBBI® Valuation Yearbook, the Duff & Phelps *2014–2017 Valuation Handbook – U.S. Guide to Cost of* Capital and starting in 2018 the online Kroll Cost of Capital Navigator. Calculations performed by Kroll, LLC.

To illustrate how the arithmetic mean is more appropriate than the geometric mean in discounting cash flows, suppose the expected return on a stock is 10% per year with a standard deviation of 20%.[1] Also assume that only two outcomes are possible each year: +30% and –10% (i.e., the mean plus or minus one standard deviation). The probability of occurrence for each outcome is equal. The growth of wealth over a two-year period is illustrated in Exhibit A11–2.

Exhibit A11–2

Growth of Wealth Example



Source: *Morningstar/Ibbotson Associates 2010 SBBI Valuation Yearbook* (Morningstar, 2010). The most common outcome of $1.17 is given by the geometric mean of 8.2%.

Compounding the possible outcomes as follows derives the geometric mean:

$$[(1 + 0.30) \times (1 - 0.10)]^{0.5} - 1 = 0.082 = 8.2\%$$

[1] This section is based on the discussion about this topic in the 2010 SBBI Valuation Edition Yearbook®. The SBBI Valuation Yearbook was discontinued in 2013; it was focused on valuation data and should not be confused with the Stocks, Bonds, Bills, and Inflation (SBBI) Yearbook published annually by D&P/Kroll since 2016 which is focused on U.S. asset class performance data. The Stocks, Bonds, Bills, and Inflation (SBBI) Yearbook is a continuation of the book previously published by Ibbotson Associates and then Morningstar under the name SBBI Classic Yearbook. The previous SBBI "Classic" Yearbook and its renamed successor the Stocks, Bonds, Bills, and Inflation (SBBI) Yearbook (the word "Classic" was removed from the name) (now published by D&P/Kroll) has been the definitive annual resource for historical U.S. capital markets performance data for over 30 years (i.e., returns, index values, and statistical analyses of U.S. large company stocks, small company stocks, long-term corporate bonds, long-term government bonds, intermediate-term government bonds, U.S. Treasury bills, and inflation from January 1926 to present (monthly)).

However, the *expected value* is predicted by compounding the arithmetic, not the geometric, mean. To illustrate this, we need to look at the probability-weighted average of all possible outcomes (probability times outcome):

(0.25 × \$1.69)	=	\$0.4225
+ (0.50 × \$1.17)	=	\$0.5850 [resulting from two occurrences of four outcomes]
+ (0.25 × \$0.81)	=	\$0.2025
Total		\$1.2100

Therefore, \$1.21 is the *probability-weighted expected value*. The rate that must be compounded to achieve the terminal value of \$1.21 after two years is 10% $[(1.21)^{0.5} - 1]$, the arithmetic mean:

$$\$1 \times (1 + 0.10)^2 = \$1.21$$

The geometric mean, when compounded, results in the median of the distribution:

$$\$1 \times (1 + 0.0.82)^2 = \$1.17$$

The arithmetic mean equates the expected future value with the present value.

The use of the arithmetic average relies on the assumptions that (1) market returns are serially independent (not correlated) and (2) the distribution of market returns is stable (not time-varying). Under these assumptions, an arithmetic average gives an unbiased estimate of expected future returns assuming expected conditions in the future are similar to conditions during the look-back period used to measure the arithmetic average. Moreover, as the number of observations increase, the resulting arithmetic average becomes a more accurate estimate.